Fuzzy Systems and Soft Computing
ISSN : 1819-4362

A CORRELATION BETWEEN $\alpha^* g^\# \psi$-CLOSED SETS IN TOPOLOGICAL SPACES AND $I\alpha^* g^\# \psi$-CLOSED SETS IN INTUITIONISTIC TOPOLOGICAL SPACES

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Abstract:
The subject of this work is Alpha star generalized hash psi closed set($\alpha^* g^\# \psi$) in topological spaces where the relation of a $\alpha^* g^\# \psi$-closed set to other generalized sets are derived. Also the characteristics of $\alpha^* g^\# \psi$-closed set is derived. In a similar way also the properties of intuitionistic ($I\alpha^* g^\# \psi$)-closed sets within intuitionistic topological spaces is derived, along with exploring their relationships with other intuitionistic generalized sets and specific characteristics of ($I\alpha^* g^\# \psi$)-closed sets. Furthermore, the paper investigates the comparability between $\alpha^* g^\# \psi$ and $I\alpha^* g^\# \psi$ in both general and intuitionistic topology.

Keywords:
Topological spaces, general topology, α-closed, g-closed, gs-closed, gp-closed, gα-closed, *gα-closed, sg-closed, αg-closed, gsp-closed, g*sp-closed, gpr-closed, ψ-closed, ψg-closed, $g^\#$-closed, $g^\#\psi$-closed, $\alpha g^\# \psi$ -closed, $\alpha^* g^\# \psi$ –closed, $I\alpha^* g^\# \psi$-closed set
MSC: 54A05, 54A10, 54C65

1. Introduction

Levine [13] developed generalized closed sets and semi-open sets in topological spaces. Njastad [9] introduced α-sets. S.P.Arya and T.Nour [13] introduced gs-closed sets. H.Maki et al.[9] developed gα-closed sets and αg-closed sets. M.Vigneshwaran and R.Devi[13] developed the idea of *gα-closed sets. gsp-closed sets and gpr-closed sets were developed by Dontchev[5] and Gnanambal[4] respectively. Veerakumar[10] introduced ψ-closed sets. Kanimozhi, Balamani and Parvathi[8] introduced $g^\#\psi$-closed sets and T.Nandhini[9] introduced $\alpha g^\#\psi$-closed sets. Zadeh [2] introduced the notion of a fuzzy set in his seminal work of 1965. The concept of fuzzy topology was subsequently developed by Chang[3] in 1968 and Azad in 1981. Atanassov[4] utilized a form of generalized fuzzy set to propose the notion of "Intuitionistic fuzzy set". Following this, Coker[4] defined intuitionistic fuzzy special sets using this framework and introduced intuitionistic fuzzy topological spaces. Coker employed fuzzy sets to define membership and non-membership concepts within intuitionistic fuzzy topological spaces, completely replacing crisp sets. Moreover, various authors have investigated and examined different types of intuitionistic topological spaces [1, 2, 3, 4, 9, 11, 14]. The intuitionistic set is given in the form A = $\langle X, A_1, A_2 \rangle$ where A_1 and A_2 are subsets of X such that their intersection $A_1 \cap A_2 = \emptyset$, A_1 is referred to as the set of members of A, while A_2 is denoted as the set of non-members of A. The objective of this study is to scrutinize the recently introduced basic properties of intuitionistic α-closed, intuitionistic semi generalized closed, intuitionistic ψ-closed, intuitionistic $g^\#\psi$-closed, intuitionistic $\alpha g^\#\psi$ -closed in intuitionistic topological spaces. The objective of this article is to develop a new group of sets named Alpha star generalized hash psi closed sets in topological spaces. In particular focusing on intuitionistic $\alpha^* g^\# \psi$ -closed sets and their characteristics.

2. Preliminaries

Definition 2.1. [6, 7, 9,13] Let P be a topological space, S ⊆T is stated as
sOS if S⊆cl(int(S)) and sCS if int(cl(S)) ⊆S.
pOS if S⊆ int(cl(S)) and pCS if cl(int(S))⊆S.
αOS if S⊆int(cl(int(S))) and αCS if cl(int(cl(S)))⊆S.

gCS if cl(S) ⊆T when S⊆T and T is open in P.
gsCS if scl(S) ⊆T when S⊆T and T is open in P.
gpCS if pcl(S) ⊆T when S⊆T and T is open in P.
gαCS if αcl(S) ⊆T when S⊆T and T is α-open in P.
*gαCS if cl(S) ⊆T when S⊆T and T is gα-open in P.
sgCS if scl(S) ⊆T when S⊆T and T is semi open in P.
αgCS if αcl(S) ⊆T when S⊆T and T is open in P.
gspCS if spcl(S) ⊆T when S⊆T and T is open in P.
g*spCS if spcl(S) ⊆T when S⊆T and T is g-open in P.
gprCS if pcl(S) ⊆T when S⊆T and T is regular open in P.
ψCS if scl(S) ⊆T when S⊆T and T is sg-open in P.
ψgCS if ψcl(S) ⊆T when S⊆T and T is open in P.
$g^{\#}$CS if cl(S) ⊆T when S⊆T and T is αg-open in P.
$g^{\#}\psi$CS if ψcl(S) ⊆T when S⊆T and T is ψ-open in P.
$\alpha g^{\#}\psi$CS if αcl(S) ⊆T when S⊆T and T is $g^{\#}\psi$-open in P.

Definition 2.2. [4] A subset τ of an intuitionistic set A in X that adheres to the following axioms constitutes an intuitionistic topology on a non-empty set:
(1) $\underset{\sim}{X}$, $\underset{\sim}{\emptyset} \in \tau$.
(2) The intersection of any two sets A_1 and A_2 in τ is also in τ.
(3) The union of any arbitrary family A_i : i $\in$ J, where each A_i is a member of τ, belongs to τ.
The pair (X, τ) is referred to as an intuitionistic topological space(ITS) and any intuitionistic set(IS) in τ is denoted as an intuitionistic open set(IOS) in X. The complement of IOS is denoted as an intuitionistic closed set(ICS) in X.

Definition 2.3. [4] Consider a non-empty set X, where an intuitionistic set A is defined as an object represented by the structure A = $\langle X, A_1, A_2 \rangle$, with A_1 and A_2 being subsets of X such that their intersection, $A_1 \cap A_2$, is the empty set ∅. Here, A_1 is referred to as the set of members of A, while A_2 is denoted as the set of non-members of A.

Definition 2.4. [4] Let X be a non empty set and let A = $\langle X, A_1, A_2 \rangle$ and B = $\langle X, B_1, B_2 \rangle$ be an intuitionistic sets on X. Then
1) A ⊆ B iff $A_1 \subseteq B_1$ and $A_2 \subseteq B_2$.
2) A ⊆ B and B ⊆ C ⟹ A ⊆ C.
3) A = B iff A ⊆ B and B ⊆ A.
4) $\bar{A} = \langle X, A_2, A_1 \rangle$
5) $A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2 \rangle$.
6) $A \cap B = \langle X, A_1 \cap B_1, A_2 \cup B_2 \rangle$.
7) $\underset{\sim}{\emptyset} = \langle X, \emptyset, X \rangle$ and $\underset{\sim}{X} = \langle X, X, \emptyset \rangle$
8) $A_i \subseteq B$ for each i ∈ J ⟹ $\cup A_i \subseteq B$.
9) $B \subseteq A_i$ for each i ∈ J ⟹ $B \subseteq \cap A_i$.
10) $(\cup Ai)^c = \cap Ai^c$ and $(\cap Ai)^c = \cup Ai^c$.
11) $((A)^c)^c = A$.
12) A - B = A ∩ B.
13) [ ]A = $\langle X, A_1, (A_1)^c \rangle$ and <>A = $\langle X, (A_2)^c, A_2 \rangle$.

Definition 2.5. [4] Let A be a subset of ITS (X, τ), then the intuitionistic interior and intuitionistic closure are defined as follows:
Iint(A) = ⋃{K : K is an IOS in X and A ⊇ K}.
Icl(A) = ⋂{K : K is an ICS in X and A ⊆ K}.

Definition 2.6. [12] Let (X, τ) be an ITS, then IS A of X is said to be an intuitionistic
- semi open(Is-open) if A ⊆ Icl(Iint(A)) and semi closed(Is-closed) if Iint(Icl(A)) ⊆ A.
- pre open(Ip-open) if A ⊆ Iint(Icl(A)) and pre closed(Ip-closed) if Icl(Iint(A)) ⊆ A.
- α-open($I\alpha$-open) if A ⊆ Iint(Icl(Iint(A))) and α-closed(Iα-closed) if Icl(Iint(Icl(A))) ⊆ A.

Definition 2.7. [4] An intuitionistic subset A of an ITS (X, τ) is referred to as an intuitionistic
- semi-generalized closed(Isg) [7] if Iscl(A) ⊆ U whenever A ⊆ U and U is intuitionistic semi open in X.

- ψ-closed($I\psi$) [10] if Iscl(A) ⊆ U whenever A ⊆ U and U is Isg-open in X.
- $g^{\#}\psi$-closed($Ig^{\#}\psi$) [8] if $I\psi$cl(A) ⊆ U whenever A ⊆ U and U is $I\psi$-open in X.
- $\alpha g^{\#}\psi$-closed($I\alpha g^{\#}\psi$) [9] if $I\alpha$cl(A) ⊆ U whenever A ⊆ U and U is $Ig^{\#}\psi$-open in X.

Notations:

$A_1 = \underset{\sim}{\emptyset} = \langle X, \emptyset, X \rangle$, $A_2 = \underset{\sim}{X} = \langle X, X, \emptyset \rangle$, $A_3 = \langle X, \{a\}, \{b\} \rangle$, $A_4 = \langle X, \{b\}, \{a\} \rangle$, $A_5 = \langle X, \{b\}, \{c\} \rangle$, $A_6 = \langle X, \{c\}, \{b\} \rangle$, $A_7 = \langle X, \{a\}, \{c\} \rangle$, $A_8 = \langle X, \{c\}, \{a\} \rangle$, $A_9 = \langle X, \{a, b\}, \{c\} \rangle$, $A_{10} = \langle X, \{c\}, \{a, b\} \rangle$, $A_{11} = \langle X, \{b, c\}, \{a\} \rangle$, $A_{12} = \langle X, \{a\}, \{b, c\} \rangle$, $A_{13} = \langle X, \{a, c\}, \{b\} \rangle$, $A_{14} = \langle X, \{b\}, \{a, c\} \rangle$, $A_{15} = \langle X, \{a\}, \emptyset \rangle$, $A_{16} = \langle X, \emptyset, \{a\} \rangle$, $A_{17} = \langle X, \{b\}, \emptyset \rangle$, $A_{18} = \langle X, \emptyset, \{b\} \rangle$, $A_{19} = \langle X, \{c\}, \emptyset \rangle$, $A_{20} = \langle X, \emptyset, \{c\} \rangle$, $A_{21} = \langle X, \{a, b\}, \emptyset \rangle$, $A_{22} = \langle X, \emptyset, \{a, b\} \rangle$, $A_{23} = \langle X, \{b, c\}, \emptyset \rangle$, $A_{24} = \langle X, \emptyset, \{b, c\} \rangle$, $A_{25} = \langle X, \{a, c\}, \emptyset \rangle$, $A_{26} = \langle X, \emptyset, \{a, c\} \rangle$, $A_{27} = \langle X, \emptyset, \emptyset \rangle$

3. Basic Properties of $\alpha^{*}g^{\#}\psi$ -closed sets

Definition 3.1: Let P be a topological space, a subset S of P is stated as $\alpha^{*}g^{\#}\psi$-closed set($\alpha^{*}g^{\#}\psi$-CS) if αcl(S) ⊆T when S⊆T and T is $\alpha g^{\#}\psi$-open set($\alpha g^{\#}\psi$-OS).

Theorem 3.2: Every closed set is $\alpha^{*}g^{\#}\psi$ -CS.

Proof: Let S⊆T, T is $\alpha g^{\#}\psi$-OS in P.
As S is closed, cl(S) = S.
But αcl(S) ⊆cl(S).
$\Longrightarrow$ αcl(S) ⊆T.
Hence S is $\alpha^{*}g^{\#}\psi$ -CS.

The below example demonstrates the preceding implication is irreversible.

Example 3.3: Let P = $\{x_1, x_2, x_3\}$, $\tau = \{P, \emptyset, \{x_1, x_3\}\}$.
$\alpha^{*}g^{\#}\psi$C(P, τ) = $\{P, \emptyset, \{x_2\}, \{x_1, x_2\}, \{x_2, x_3\}\}$.
$\{x_1, x_2\}$ is $\alpha^{*}g^{\#}\psi$-CS of (P, τ) although not closed in (P, τ).

Theorem 3.4: Every α-CS is $\alpha^{*}g^{\#}\psi$ -CS.

Proof: Let S⊆T, T is $\alpha g^{\#}\psi$-OS in P.
As S is α-CS, αcl(S) = S.
But αcl(S) ⊆T.
Hence S is $\alpha^{*}g^{\#}\psi$ -CS.

The below example demonstrates the preceding implication is irreversible.

Example 3.5: Let P = $\{x_1, x_2, x_3\}$, $\tau = \{P, \emptyset, \{x_2, x_3\}\}$.
αC(P, τ) = $\{P, \emptyset, \{x_1\}\}$.
$\alpha^{*}g^{\#}\psi$ C(P, τ) = $\{P, \emptyset, \{x_1\}, \{x_1, x_2\}, \{x_1, x_3\}\}$.
$\{x_1, x_2\}$ is $\alpha^{*}g^{\#}\psi$ -CS of (P, τ) although not α-CS in (P, τ).

Remark 3.6:

Every *gα-CS, $g^{\#}$-CS, $\alpha g^{\#}\psi$ -CS is $\alpha^{*}g^{\#}\psi$-CS.

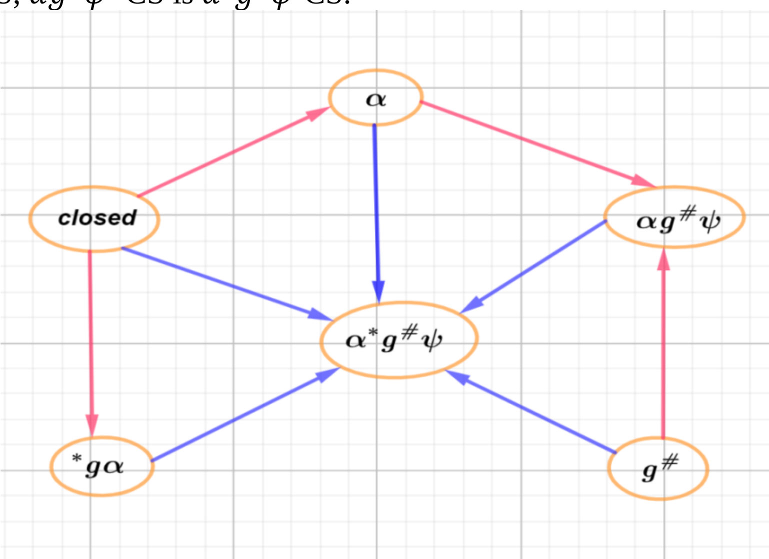

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Figure 1 : A ⟵ B

Theorem 3.6: Every $\alpha^* g^\# \psi$ -CS is gs-CS.

Proof: Let S⊆T, T is OS in P.

As every OS is $\alpha g^\# \psi$ -OS, T is $\alpha g^\# \psi$ -OS.

Here S is $\alpha^* g^\# \psi$-CS,

$\Longrightarrow$ αcl(S) ⊆T.

Also scl(S) ⊆ αcl(S), then scl(S) ⊆T.

Hence S is gs-CS.

The below example demonstrates the preceding implication is irreversible.

Example 3.7: Let P = $\{x_1, x_2, x_3\}$, $\tau = \{P, \emptyset, \{x_2\}, \{x_1, x_2\}\}$.

GSC(P, τ) = $\{P, \emptyset, \{x_1\}, \{x_3\}, \{x_2, x_3\}, \{x_1, x_3\}\}$.

$\alpha^* g^\# \psi$C(P, τ) = $\{P, \emptyset, \{x_1\}, \{x_3\}, \{x_1, x_3\}\}$.

$\{x_2, x_3\}$ is gs-CS of (P, τ) although not $\alpha^* g^\# \psi$-CS in (P, τ).

Theorem 3.8: Every $\alpha^* g^\# \psi$-CS is gp-CS.

Proof: Let S⊆T,T is OS in P.

As every OS is $\alpha g^\# \psi$ -OS, T is $\alpha g^\# \psi$ -OS.

Here S is $\alpha^* g^\# \psi$ -CS.

$\Longrightarrow$ αcl(S) ⊆T.

Also, pcl(S) ⊆cl(S).

$\Longrightarrow$pcl(S) ⊆T.

Hence S is gp-CS.

The below example demonstrates the preceding implication is irreversible.

Example 3.9: Let P = $\{x_1, x_2, x_3\}$, $\tau = \{P, \emptyset, \{x_2, x_3\}\}$.

GPC(P, τ) = $\{P, \emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}\}$.

$\alpha^* g^\# \psi$C(P, τ) = $\{P, \emptyset, \{x_1\}, \{x_1, x_2\}, \{x_1, x_3\}\}$.

$\{x_2\}$ is gp-CS of (P, τ) although not $\alpha^* g^\# \psi$-CS in (P, τ).

Remark 3.9:

Every $\alpha^* g^\# \psi$ -CS is gs-CS, gp-CS, sg-CS, αg-CS, gsp-CS, g*sp-CS, gpr-CS, ψg-CS, $g^\# \psi$-CS

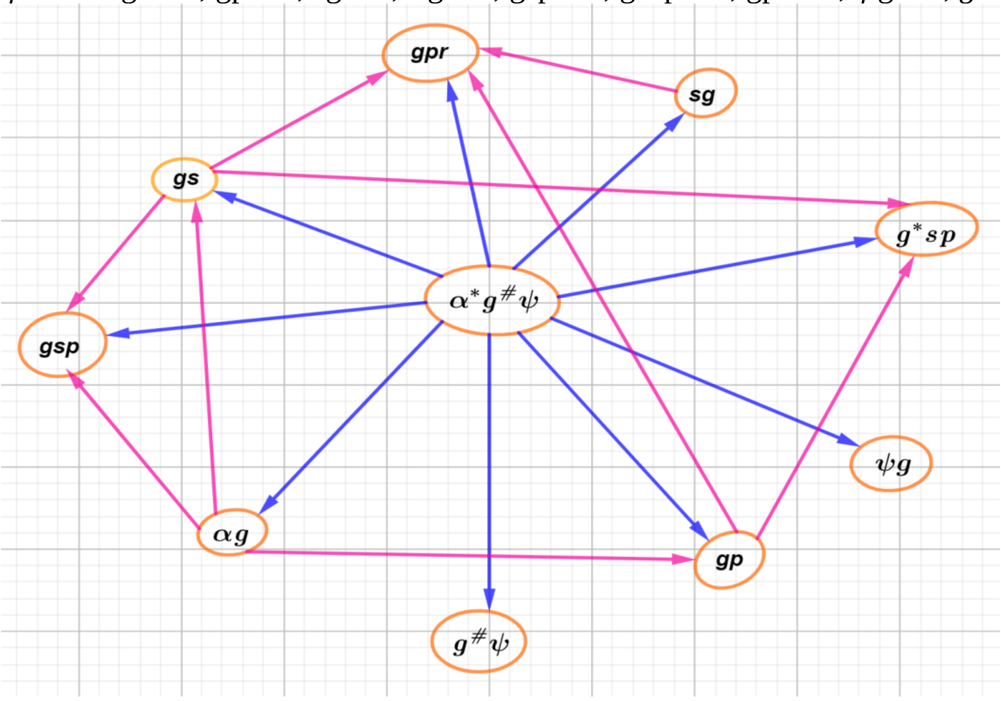

Figure 2 : A ⟶ B

4. Characteristics of $\alpha^* g^\# \psi$-closed sets

Remark 4.1: $\alpha^* g^\# \psi$-CS is independent of semi-CS and ψ-CS.

It can be shown by the following examples.

Example 4.2: Let P = $\{x_1, x_2, x_3\}$, $\tau = \{P, \emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$.

SC(P, τ) = $\{P, \emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_2, x_3\}, \{x_1, x_3\}\}$ = ψC(P, τ).
$\alpha^* g^\# \psi$C(P, τ) = $\{P, \emptyset, \{x_3\}, \{x_2, x_3\}, \{x_1, x_3\}\}$.
Here $\{x_2\}$ is semi-CS and ψ-CS of (P, τ) although not $\alpha^* g^\# \psi$-CS in (P, τ).
Let P = $\{x_1, x_2, x_3\}$, τ = $\{P, \emptyset, \{x_3\}, \{x_1, x_2\}\}$.
SC(P, τ) = $\{P, \emptyset, \{x_3\}, \{x_1, x_2\}\}$. = ψC(P, τ).
$\alpha^* g^\# \psi$C(P, τ) = $\{P, \emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}\}$.
Here $\{x_1\}$ is $\alpha^* g^\# \psi$-CS of (P, τ) although not semi-CS and ψ-CS in (P, τ).

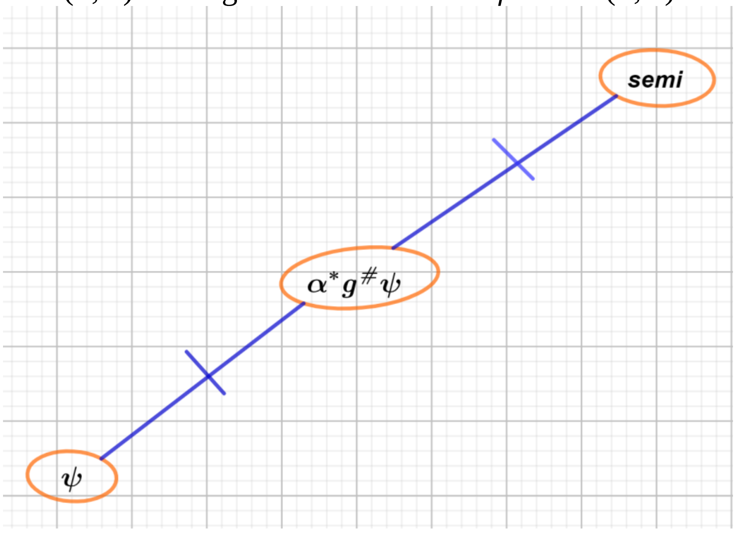

Figure 3: A ⇎ B

Theorem 4.3: The intersection of two $\alpha^* g^\# \psi$-CS is again $\alpha^* g^\# \psi$-CS.
Proof : Let X and Y be $\alpha^* g^\# \psi$-CS and X∩Y⊆T where T is $\alpha g^\# \psi$-OS.
As X and Y are $\alpha^* g^\# \psi$-CS, αcl(X) ⊆T and αcl(Y) ⊆T.
Also αcl(X ∩Y)= αcl(X)∩αcl(Y) ⊆T.
⟹ αcl(X∩Y) ⊆T where X∩Y is $\alpha^* g^\# \psi$-CS.
Hence, the intersection of two $\alpha^* g^\# \psi$-CS is again $\alpha^* g^\# \psi$-CS.
Theorem 4.4: The union of two $\alpha^* g^\# \psi$-CS is again $\alpha^* g^\# \psi$-CS.
Proof : Let X and Y be $\alpha^* g^\# \psi$-CS and X∪Y⊆T where T is $\alpha g^\# \psi$-OS.
As X and Y are $\alpha^* g^\# \psi$-CS, αcl(X) ⊆T and αcl(Y) ⊆T.
Also αcl(X∪Y)= αcl(X)∪αcl(Y) ⊆T.
⟹ αcl(X∪Y) ⊆T where X∪Y is $\alpha^* g^\# \psi$-CS.
Hence, the union of two $\alpha^* g^\# \psi$-CS is again $\alpha^* g^\# \psi$-CS.
Theorem 4.5: Let X be an OS and Y be an $\alpha^* g^\# \psi$-OS, then X∪Y is $\alpha^* g^\# \psi$-OS.
Proof : Let X be an OS of (P, τ) and Y be an $\alpha^* g^\# \psi$-OS of (P, τ)
As every OS is $\alpha^* g^\# \psi$-OS, X is $\alpha^* g^\# \psi$-OS.
Hence X∪Y is $\alpha^* g^\# \psi$-OS.
Since union of two $\alpha^* g^\# \psi$-OS is again $\alpha^* g^\# \psi$-OS.
Theorem 4.6: Let X be an $\alpha^* g^\# \psi$-CS of (P, τ) iff αcl(X)- X ⊈ any non-empty $\alpha g^\# \psi$-CS.
Proof :
Necessary Part: Assume X is $\alpha^* g^\# \psi$-CS and F be a non empty $\alpha g^\# \psi$-CS with F ⊆ αcl(X)- X.
Then X ⊆ P- F
⟹ αcl(X) ⊆ P - F.
Hence, F ⊆ P- αcl(X), which contradicts.
Sufficient Part : Assume X is a subset of (P, τ) such that αcl(X)- X ⊈ any non- empty $\alpha g^\# \psi$-CS.
Let T be an $\alpha g^\# \psi$-OS in (P, τ) such that X⊆T.
If αcl(X) ⊆T, then αcl(X) ∩ C(T) ≠ ∅.
Then ∅ ≠ αcl(X) ∩C(T) is an $\alpha g^\# \psi$-CS of (P, τ), since the intersection of two $\alpha g^\# \psi$-CS is again $\alpha g^\# \psi$-CS.
Theorem 4.7: If X is $\alpha^* g^\# \psi$-CS and X⊆Y ⊆ αcl(X), then Y is $\alpha^* g^\# \psi$-CS.

Proof: Let T be an $\alpha g^{\#}\psi$-OS of (P, τ)) such that Y⊆T.
⟹ X⊆T.
As X is $\alpha^{*}g^{\#}\psi$-CS and αcl(X) ⊆T,
Then αcl(Y)⊆ αcl(αcl(X)) = αcl(X) ⊆T.
Hence, Y is $\alpha^{*}g^{\#}\psi$-CS of (P, τ).
Diagram representation:

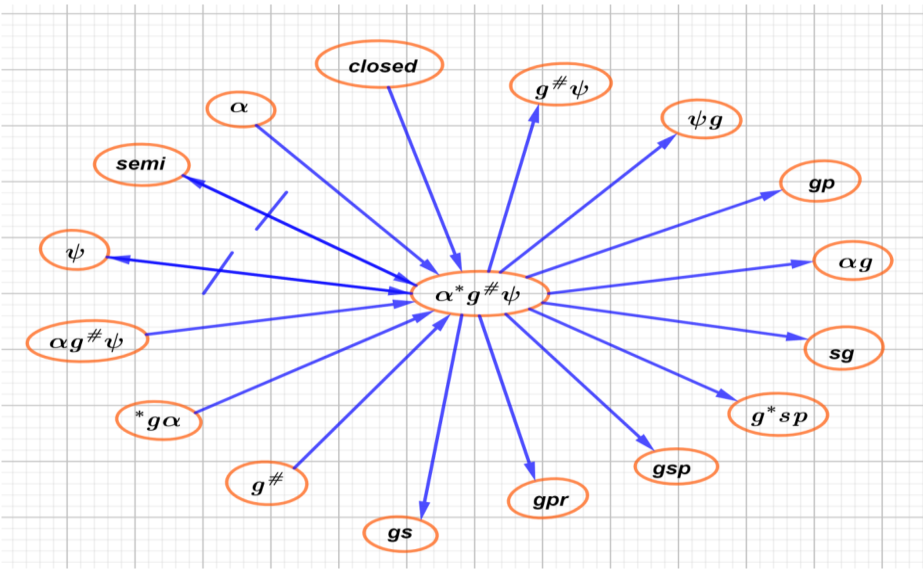

Figure 4 : A ⟶ B, A ⟵ B and A ↮ B

Remark 4.8: The diagram below illustrates the connections delineated between $\alpha^{*}g^{\#}\psi$-closed sets and various other sets in the aforementioned theorems.
A →B represents A implies B but not conversely and
A ↮ B represents A and B are mutually independent.

5. Basic properties of $I\alpha^{*}g^{\#}\psi$-closed sets

Definition 5.1. Let (X, τ) be an ITS, a subset A is said to be an $I\alpha^{*}g^{\#}\psi$-closed set(CS) if $I\alpha$cl(A) ⊆ U whenever A ⊆U and U is $I\alpha g^{\#}\psi$-open set(OS) in X.
Theorem 5.2. In ITS every intuitionistic closed set is $I\alpha^{*}g^{\#}\psi$-CS.
Proof : Consider (X, τ) as an ITS, with A representing an ICS.
Suppose A ⊆U, where U is an $I\alpha g^{\#}\psi$-OS in X.
Given that A is ICS, it implies Icl(A) = A.
However, we also know that $I\alpha$cl(A) ⊆ Icl(A).
This implies $I\alpha$cl(A) ⊆ U, thereby establishing that A is an $I\alpha^{*}g^{\#}\psi$-CS.
The subsequent example provided illustrates that this implication is not reversible.
Example 5.3. Concede X = {a,b,c}, τ = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_3}, τ^c= { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_4},
$I\alpha^{*}g^{\#}\psi$C(X, τ) = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_4 , A_{11}, A_{14}, A_{16}, A_{22}, A_{23}, A_{26}}.
Thus {A_{11}, A_{14}, A_{16}, A_{22}, A_{23}, A_{26}} are $I\alpha^{*}g^{\#}\psi$ -CS of (X, τ) although not ICS in (X, τ).
Theorem 5.4. In ITS every $I\alpha$-CS is $I\alpha^{*}g^{\#}\psi$-CS.
Proof : Consider (X, τ) as an ITS, with A representing an ICS.
Suppose A⊆ U, where U represents an $I\alpha g^{\#}\psi$-OS in X.
Given that A is an $I\alpha$-CS, it follows that $I\alpha$cl(A) = A.
However, we observe that $I\alpha$cl(A) ⊆ U.
Therefore, A qualifies as an $I\alpha^{*}g^{\#}\psi$ -CS.
The forthcoming example illustrates that this implication is not reversible.
Example 5.5. Concede X = {a,b,c}, τ = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{16}, A_{23}}, τ^c = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{15}, A_{24}},
$I\alpha$C(X, τ) = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{15}, A_{24}}.
$I\alpha^{*}g^{\#}\psi$ C(X, τ) = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_3, A_7, A_9, A_{12} , A_{13}, A_{15}, A_{21}, A_{24}, A_{25}}.

Thus {A_3, A_7, A_9, A_{12}, A_{13}, A_{15}, A_{21}, A_{24}, A_{25}} are $I\alpha^* g^\# \psi$-closed sets of (X, τ) although not $I\alpha$-CS in (X, τ).
Theorem 5.6. In ITS every $I\alpha^* g^\# \psi$ -CS is Isg-CS.
Proof : Consider (X, τ) as an ITS, and let A be an ICS.
Suppose A ⊆U, where U represents an intuitionistic semi-OS in X.
Given that every intuitionistic semi-OS is also an $I\alpha g^\# \psi$-OS, we conclude that U is an $I\alpha g^\# \psi$ -OS.
Since A is $I\alpha^* g^\# \psi$-CS, it implies $I\alpha$cl(A) ⊆ U.
Moreover, Iscl(A) ⊆ $I\alpha$cl(A).
Therefore, Iscl(A) ⊆ U, establishing that A is an Isg-CS.
The forthcoming example illustrates that this implication is not reversible.
Example 5.7. Concede X = {a,b,c}, τ = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{20}, A_{21}}, τ^c = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{19}, A_{22}},
IsgC(X, τ) = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_6, A_8, A_{10}, A_{11}, A_{13}, A_{16}, A_{18}, A_{19}, A_{20}, A_{22}, A_{23}, A_{24}, A_{25}, A_{26}, A_{27}}.
$I\alpha^* g^\# \psi$ C(X, τ) = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_6, A_8, A_{10}, A_{11}, A_{13}, A_{19}, A_{22}, A_{23}, A_{25}}.
Thus { A_{16}, A_{18}, A_{20}, A_{24}, A_{26}, A_{27}} are Isg-CS of (X, τ) although not $I\alpha^* g^\# \psi$-CS in (X, τ).
Theorem 5.8. In ITS every $I\alpha^* g^\# \psi$ -CS is $Ig^\# \psi$-CS.
Proof : Consider (X, τ) as an ITS, where A represents an ICS.
Suppose A ⊆ U, where U is an $I\psi$-OS in X.
Since every $I\psi$-OS is also an $I\alpha g^\# \psi$-OS, it follows that U is an $I\alpha g^\# \psi$-OS.
Given that A is an $I\alpha^* g^\# \psi$-CS, we have $I\alpha$cl(A) ⊆ U.
However, we know that $I\psi$cl(A) ⊆ $I\alpha$cl(A).
This implies $I\psi$cl(A) ⊆ U, thereby establishing that A is an $Ig^\# \psi$-CS.
The following example demonstrates that this implication is not reversible.
Example 5.9. Concede X = {a,b,c}, τ = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{15}, A_{24}}, τ^c = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{16}, A_{23}},
$Ig^\# \psi$C(X, τ) = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_4, A_5, A_6, A_8, A_{10}, A_{11}, A_{14}, A_{16}, A_{17}, A_{18}, A_{19}, A_{20}, A_{22}, A_{23}, A_{24}, A_{26}, A_{27}}.
$I\alpha^* g^\# \psi$ C(X, τ) = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{11}, A_{16}, A_{22}, A_{23}, A_{26}}.
Thus { A_4, A_5, A_6, A_8, A_{10}, A_{14}, A_{17}, A_{18}, A_{19}, A_{20}, A_{24}, A_{27}} are $Ig^\# \psi$-CS of (X, τ) although not $I\alpha^* g^\# \psi$-CS in (X, τ).
Theorem 5.10. In ITS every $I\alpha g^\# \psi$-CS is $I\alpha^* g^\# \psi$ -CS.
Proof : Consider (X, τ) as an ITS, where A denotes an ICS.
Suppose A ⊆ U, where U is an $I\alpha g^\# \psi$-OS in X.
Since every $I\alpha g^\# \psi$-OS is also an $Ig^\# \psi$-OS, it implies that U is an $Ig^\# \psi$-OS.
Given that A is an $I\alpha g^\# \psi$-CS, we have $I\alpha$cl(A) ⊆ U.
Thus, A qualifies as an $I\alpha^* g^\# \psi$ -CS.
The example provided below demonstrates that this implication is not reversible.
Example 5.11. Concede X = {a,b,c}, τ = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{18}, A_{25}}, τ^c = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_{17}, A_{26}},
$I\alpha g^\# \psi$C(X, τ) = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_4, A_5, A_{14}, A_{17}, A_{26}}.
$I\alpha^* g^\# \psi$ C(X, τ) = { $\underset{\sim}{X}$, $\underset{\sim}{\emptyset}$, A_4, A_5, A_9, A_{11}, A_{14}, A_{17}, A_{21}, A_{23}, A_{26}}.
Thus { A_9, A_{11}, A_{21}, A_{23}} are $I\alpha^* g^\# \psi$-CS of (X, τ) although not $I\alpha g^\# \psi$-CS in (X, τ).

6. Characteristics of $I\alpha^* g^\# \psi$ -closed sets

Theorem 6.1. The intersection of two $I\alpha^* g^\# \psi$-CS is again $I\alpha^* g^\# \psi$-CS.
Proof : Assume A and B as $I\alpha^* g^\# \psi$-CS, and consider their intersection A ∩ B ⊆ U, where U is an $I\alpha g^\# \psi$ -OS.
Given that both A and B are $I\alpha^* g^\# \psi$-CS, it follows that $I\alpha$cl(A) ⊆ U and $I\alpha$cl(B) ⊆U.
Moreover, $I\alpha$cl(A ∩ B) = $I\alpha$cl(A) ∩ $I\alpha$cl(B) ⊆ U.
Thus, $I\alpha$cl(A ∩ B) ⊆ U, indicating that A ∩ B is an $I\alpha^* g^\# \psi$-CS.
Therefore the intersection of two $I\alpha^* g^\# \psi$-CS remains an $I\alpha^* g^\# \psi$-CS.
Theorem 6.2. The union of two $I\alpha^* g^\# \psi$ -CS is again $I\alpha^* g^\# \psi$ -CS.
Proof : Suppose A and B are sets ICS under the topology induced by $I\alpha^* g^\# \psi$, with A∪ B ⊆ U, where U is an IOS under $I\alpha g^\# \psi$.
Since both A and B are CS under $I\alpha^* g^\# \psi$, it follows that $I\alpha$cl(A) ⊆ U and $I\alpha$cl(B) ⊆ U.

Moreover, $I\alpha$cl(A ∪ B)= $I\alpha$cl(A) ∪ $I\alpha$cl(B) ⊆ U.
⇒ $I\alpha$cl(A ∪ B) ⊆ U indicating that A ∪ B is an $I\alpha^*g^\#\psi$-CS.
Thus, the union of two CS under $I\alpha^*g^\#\psi$ remains CS under $I\alpha^*g^\#\psi$.
Theorem 6.3. If A is an $I\alpha^*g^\#\psi$ -CS of an ITS (X, τ) and A ⊆ B ⊆ $I\alpha$cl(A), then B is $I\alpha^*g^\#\psi$ -CS in X.
Proof : Consider A as an $I\alpha^*g^\#\psi$ -CS of an ITS (X, τ) and A ⊆ B ⊆ $I\alpha$cl(A).
Suppose U is an $I\alpha g^\#\psi$ -OS of (X, τ) such that B ⊆ U.
Then it follows that A ⊆ U.
As A is $I\alpha^*g^\#\psi$ -CS and $I\alpha$cl(A) ⊆ U,
Moreover $I\alpha$cl(B) ⊆ $I\alpha$cl($I\alpha$cl(A)) = $I\alpha$cl(A) ⊆ U.
Hence B is $I\alpha^*g^\#\psi$ -CS of (X, τ).
Theorem 6.4. Let A be an IOS and B be an $I\alpha^*g^\#\psi$-OS, then A ∪ B is $I\alpha^*g^\#\psi$-OS.
Proof : Let A be an IOS of (X, τ) and B be an $I\alpha^*g^\#\psi$-OS of (X, τ).
As every IOS is $I\alpha^*g^\#\psi$-OS, A is $I\alpha^*g^\#\psi$-OS.
Hence A ∪ B is $I\alpha^*g^\#\psi$-OS.
Since union of two $I\alpha^*g^\#\psi$-OS is again $I\alpha^*g^\#\psi$-OS.
Theorem 6.5. Let A be an $I\alpha^*g^\#\psi$ -CS of (X, τ) iff $I\alpha$cl(A)-A ⊈ any non-empty $I\alpha g^\#\psi$-CS.
Proof :
Necessary Part : Assume A is an $I\alpha^*g^\#\psi$-CS and F be a non empty $I\alpha g^\#\psi$-CS with F ⊆ $I\alpha$cl(A)-A.
Then A ⊆ X-F
⟹ $I\alpha$cl(A) ⊆ X-F.
Hence, F ⊆ X- $I\alpha$cl(A), which contradicts.
Sufficient Part : Assume A is a subset of (X, τ) such that $I\alpha$cl(A)-A ⊈ any non-empty $I\alpha^*g^\#\psi$-CS.
Let U be an $I\alpha g^\#\psi$-OS in (X, τ) such that A ⊆ U.
If $I\alpha$cl(A) ⊆ U, then $I\alpha$cl(A) ∩ C(U) ≠ ∅.
Then ∅ ≠ $I\alpha$cl(A) ∩ C(U) is an $I\alpha g^\#\psi$-CS of (X, τ), since the intersection of two $I\alpha g^\#\psi$-CS is again $I\alpha g^\#\psi$-CS.
Remark 6.6. The diagram illustrates the connections delineated between $I\alpha^*g^\#\psi$ -closed sets and other generalized intuitionistic sets as described in the aforementioned theorems.
The notation A → B signifies that A implies B, but not necessarily vice versa.

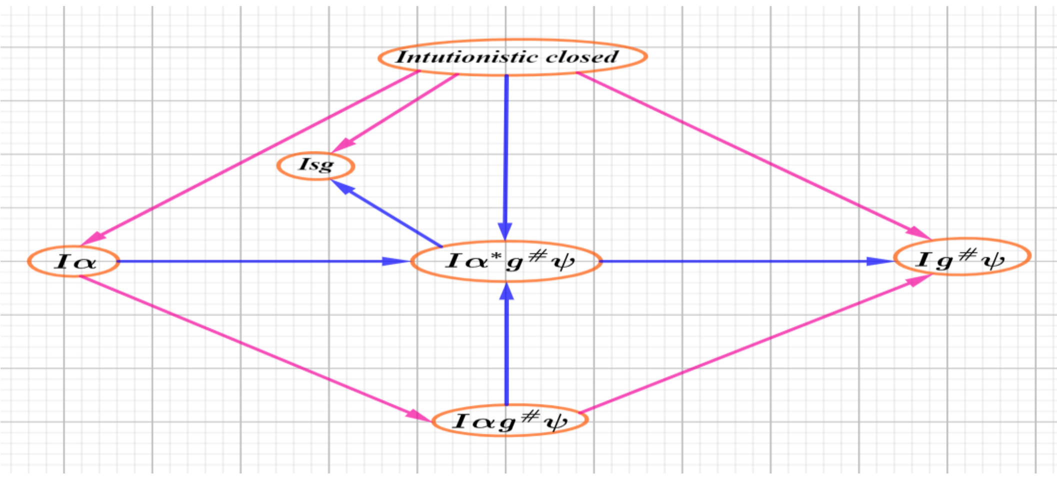

Figure 5 : A ⟶ B

5. Conclusion
In this paper, we have explored the concept of closed sets within the framework of both general and intuitionistic topology. Through our examination, we have elucidated the fundamental properties and characteristics that define closed sets in different contexts, including topological spaces and generalized intuitionistic spaces. Our investigation has revealed that intuitionistic closed sets provide a constructive approach to understanding continuity, convergence, and topological structure in intuitionistic spaces. Additionally, future research could focus on the development of computational

techniques and algorithms for reasoning about intuitionistic closed sets. By leveraging advances in computational logic and proof theory, researchers can explore automated methods for analyzing and manipulating intuitionistic closed sets, facilitating the development of formal reasoning systems and constructive proof assistants.

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